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TRUTH APPROXIMATION BY CONCRETIZATION IN CAPITAL STRUCTURE THEORY

ABSTRACT. This paper supplies a structuralist reconstruction of the Modigliani-Miller theory and shows that the economic literature following their results reports on research with an implicit strategy to come "closer-to-the-truth" in the modern technical sense in philosophy of science.

> "These and other drastic simplifications have been necessary in order to come to grips with the problem at all ... Having served this purpose, they can now be relaxed in the direction of greater realism and relevance, a task in which we hope others interested in this area will wish to share"[2]

Introduction

In 1982 one of the authors published a structuralist theory of truth approximation (Kuipers 1982). The main ideas were the following. According to the naive structuralist approach theories can be represented as sets of structures. Point of departure is a set *Mp* of structures of a certain similarity type, called the potential models or conceptual possibilities. The target of theorizing is some unknown subset. In proper empirical contexts the target set is the set of empirical possibilities. In other contexts (such as applied mathematics and pure economic theory) the target set may be the set of conceptual possibilities in which a certain interesting theorem is valid. A theory is now conceived as the combination of some subset *X* of *Mp*, together with the claim that *X* in fact coincides with the target set.

The core of the naive structuralist theory of truthlikeness is based on the following ternary relation of theorylikeness: theory *Y* is closer to theory *Z* than theory *X* is to *Z* if *Y-Z* is a subset of *X-Z* and *Z-Y* is a

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[1] We are indebted to Wolfgang Balzer for helpful comments on an earlier draft of this paper.
[2] Modigliani and Miller (1958) p. 296.

subset of Z-X.[3] This transforms into naive truthlikeness by substituting for Z the target set of empirical possibilities.

The naive definition has many attractive aspects, among others, it has not the famous shortcomings of Popper's original definition of truthlikeness. However, it is also immediately clear that the naive theory cannot cover cases were similarity between structures plays a crucial role, as is normal, in particular in quantitative cases. One important type concerns theories which are improved by concretization. The theory of gases of Van der Waals is an improvement of the so-called ideal theory of gases, by accounting quantitative factors related to the volume and mutual attraction of molecules. Hence, a satisfactory definition of refined truthlikeness, i.e. on the basis of an underlying structurelikeness relation, should contain concretization to the true theory as a special case and should reduce to the naive one when the notion of structurelikeness is trivial.

After a first failing attempt at a refined definition (Kuipers 1987), recently has been construed a satisfactory refined definition (Kuipers 1992).[4] In the final section of that paper it was shown that the refined definition could deal with concretization in general and the Van der Waals example in particular. Moreover, it was indicated that it could deal with elementary and complex forms of concretization in the context of validity research. The main point of the present paper is to show under what conditions and in what sense the theory of Kraus and Litzenberger (1973) about the capital structure of firms is a concretization of the theory of Modigliani and Miller bringing us closer to a provable interesting truth about the actual financing behavior of firms.

In Section 1 we present an informal exposition of the crucial results in capital structure theory. The first author was also the one who suggested to elaborate this example as a possible case of truth approximation by concretization. Although it would have been more easy to take an example from general economics, e.g. the case of international trade (Hamminga 1983), an example from financial economics has the additional advantage that it illustrates the formal kinship of theory structure and development in business economics and other applications of neoclassical equilibrium theory. In Section 2 the second author gives a structuralist reconstruction of the relevant theories. This reconstruction is required in order to be able in Section 3 to deal with the relevant theories as sets of structures and to enable the easy checking of some crucial logico-mathematical claims. An independent merit of Section 2

[3] Where "Y-Z" denotes the set of all elements of Y that are not in Z.

[4] For a much shorter version vid. Kuipers (1992a).

is that it provides an exemplification of the claim that not only theories in general economics (e.g. Janssen 1989, Janssen and Kuipers 1989), but also in financial economics can be made transparent by structuralist reconstruction. Finally, in Section 3 the third author explains in some detail what the example illustrates: economists attempt to approach interesting truths by concretization, and they may well succeed in this. In general, truthlikeness research attempts to explicate what working scientists always implicitly assume, but most philosophers like to make suspect.

Section 1

The First Three Steps in Capital Structure Theory

1.1. Introduction

"How do firms choose their capital structure?" is one of the most important issues in corporate finance – and one of the most complex. The "capital structure" is the ratio between debt (money borrowed by a firm at a fixed interest rate), and equity (money invested in the firm by shareholders that own the firm, have full posession of its assets and profits, and thus are the residual claimants)[5] Corporate finance is part (the other part being investment theory) of financial economics, which is a branch of applied micro economics and therefore based on neoclassical utility theory. In additon to standard utility theory assumptions, in corporate finance it is assumed that the goal of a company is to maximize shareholders' wealth (= utility) i.e. maximize the market value of equity.[6,7] Given the above mentioned goal of the firm, the answer to the question "How do firms choose their capital structure?" can be rephrased as "Which capital structure maximizes the value of the firm?" or "Can the value of the assets be increased by an optimal financing policy of the firm?".

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[5] The term 'capital structure', more broadly defined, refers to the way in which a firm finances its assets, it not only denotes the relative quantities of debt and equity, but also warrants, trade credit, convertibles, leases, etc. For simplicity we shall assume that the capital structure only consists of debt (money borrowed from banks or bondholders) and equity (stocks or shares).

[6] For reasons of brevity we will hereafter speak of "value" in stead of "market value".

[7] By definition, the value of the firm equals the value of equity plus the value of debt. And, since the value of the debt does not change because it is assumed to be risk free, maximizing the value of the equity (= shareholders' wealth) is equal to maximizing the value of the whole firm.

In 1958 Franco Modigliani and Merton Miller wrote their seminal paper on the issue of the optimal capital structure. Although for some decades the paper has been the subject of intense scrutiny and often bitter controversy, and Franco Modigliani once stated that "I must confess that ... my two articles with Miller on corporate finance are written with tongue in cheek, to really make fun of my colleagues." (Klamer 1983, p. 125). Most of these controversies can now be regarded as settled: the essential results of the paper have overcome. Moreover, the results, often called the MM propositions, have spread beyond corporate finance to the fields of money and banking, fiscal policy and international finance (for examples, see Miller 1988). And finally, the scientific community has recognized the importance of the paper by awarding both authors the Nobel Prize in economics (Modigliani in 1984 and Miller in 1990).

Proposition I of MM has become the first step in capital structure theory and is sometimes called the 'nothing matters' or 'irrelevance' theorem. It states that, as an implication of equilibrium in perfect capital markets, the value of a firm is independent of its capital structure (that is, its debt/equity ratio). In proving this proposition they used a then-novel arbitrage argument, which is now common throughout finance. The second step was also made by MM, first in their 1958 paper, but corrected in Modigliani and Miller (1963). It says that, when corporate taxes – interest payments are tax deductible – are introduced in the model, 100% debt financing is optimal. The intuition behind this being that the more debt, the less taxes a firm pays, and therefore the more money – thus value – is left for the financeers (shareholders and debtholders) of the company. This result was extremely puzzling, since in real world one never observes firms with 100% debt financing. The third step in capital structure theory was first suggested by Baxter (1976) and later formalized by others. Now, bankruptcy costs are introduced. These costs consist of payments that must be made to third parties other than bond- or shareholders when the firm goes bankrupt, such as trustee fees, legal fees, costs of reorganisation, etc. These "dead weight" losses associated with bankruptcy cause the value of the firm to be less than it would have been otherwise, namely the value based on the expected cash flows from operations. And since the change of going bankrupt is higher when a firm is financed with more debt, there are costs involved with debt financing. The tradeoff between the tax advantage of debt and bankruptcy costs associated with debt results in an optimal capital structure, the so called balancing theorem.

To summarize, in a perfect world – without taxes of bankruptcy costs – the debt/equity ratio is irrelevant for the value of the firm (theorem A). When the imperfection of corporate taxes is introduced, 100% debt financing is optimal, i.e. maximizes the value of the firm (theorem B). Finally, when also bankruptcy costs are taken into consideration, there is a cost to debt financing and an interior solution for the optimal capital structure emerges; a debt/equity ratio somewhere between 0% and 100% maximizes the value of the firm (theorem C). Theoretically, it would also be possible to consider a world with only the imperfection of bankruptcy cost (and no corporate taxes), in which case 100% equity financing would be optimal (theorem D).

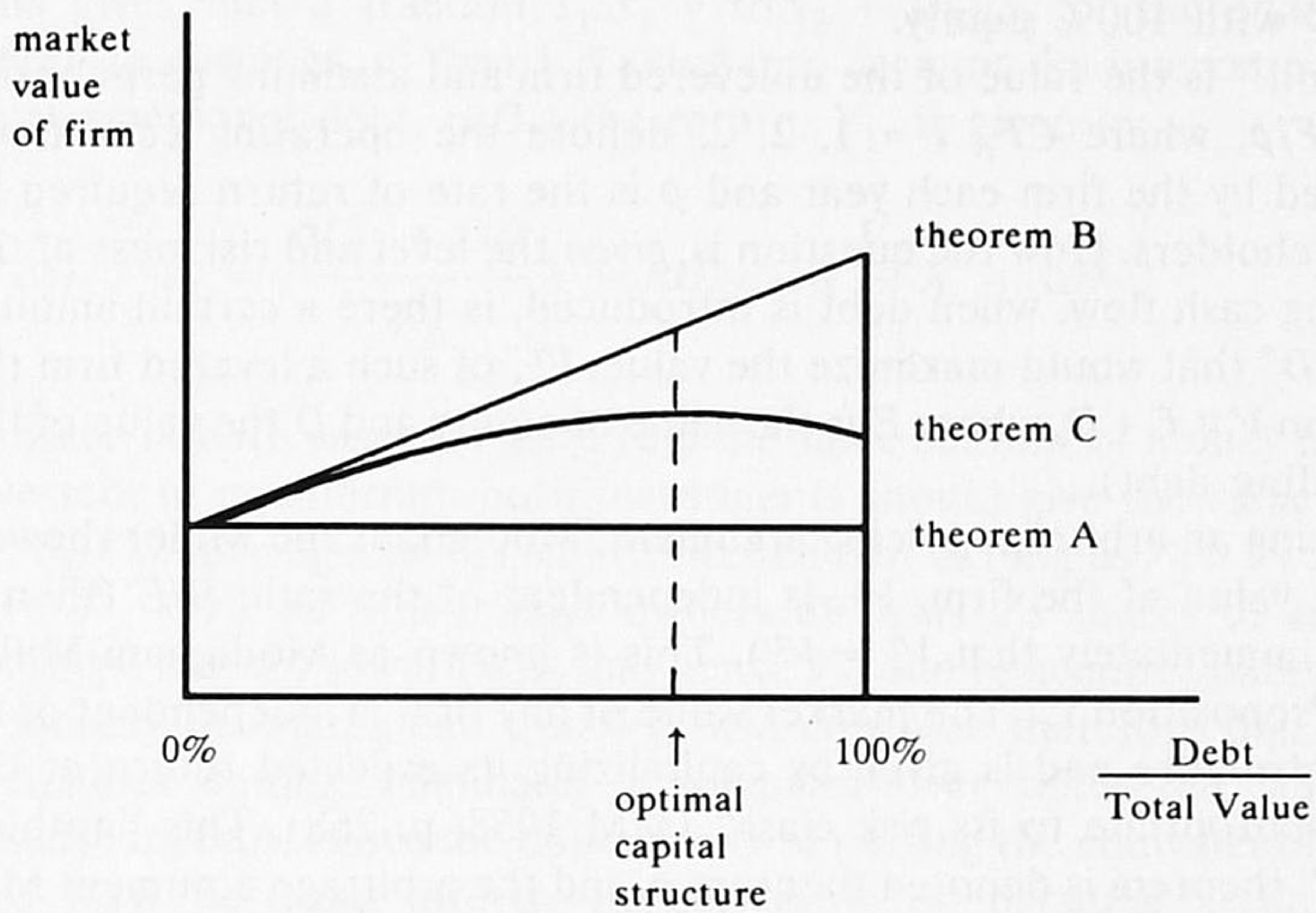

Figure 1. Optimal capital structure

Now, let us go more into the details of the famous MM "Proposition I"

1.2. Modigliani and Miller's "nothing matters", theorem A

The value of the firm equals the net present value, *NPV*, of future expected cash flows (that is: net receipts of money after deduction of all cost exept interest payments). The *NPV* of a τ-period 'project' can be written as

$$NPV = \sum_{t=0}^{\tau} \frac{CF_t}{(1+r)^t} \, ,$$

where CF_t, is the cash flow on time t and r is the required rate of return or discount rate. r is determined by the time value of money (the risk free interest rate[8]) and the riskiness of the expected cash flows, CF. In case of a perpetuity (i.e. an infinite, constant stream of cash flows) the NPV can be written as

$$NPV = \frac{CF}{r}.^{9}$$

An unlevered firm is a firm that has no debt, in other words, it is financed with 100% equity.

When V^U is the value of the unlevered firm and assuming perpetuities $V^u = CF/\rho$, where CF_t, $t = 1, 2, \ldots$ denote the operating cash flows generated by the firm each year and ρ is the rate of return required by the shareholders. How the question is, *given* the level and riskiness of the operating cash flow, when debt is introduced, is there a certain amount of debt D^* that would maximize the value, V^L, of such a levered firm (by definition $V \equiv E + D$, where E is the value of equity and D the value of the outstanding debt).

By using an arbitrage pricing argument, Modigliani and Miller showed that the value of the firm, V^L, is independent of the ratio D/E (then it follows immediately that $V^L = V^U$). This is known as Modigliani-Miller (MM) Proposition I: "The market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ appropriate to its risk class" (MM 1958, p. 268). This "nothing matters" theorem is denoted theorem A and the arbitrage argument MM used to prove their Proposition I can be summarized as follows. Using the same notation as MM, V_1 is an unlevered firm (only equity) and V_2 has some debt in its capital structure, X is the total annual return (cash flow), r is the interest charge, D is the market value of debt and S is the market value of equity.

Then: $V_1 = S_1$ and the income available for the stockholders equals X_1

$V_2 = S_2 + D_2$ and the income available for the stockholders equals $X_2 - rD_2$

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[8] This is a remuneration for providing money without any risk, the value of which is determined by inflation, time preference of consumption and other factors.

[9] This can be proven as follows. Let $CF/(1+r) = a$ and $1/(1+r) = x$. We now have $NPV = a(1+x+x^2+\ldots)$ (1). Multiplying by x gives $xNPV = ax+ax^2+\ldots$ (2). Substracting (2) from (1) gives $NPV(1-x) = a$. Now substitute for x and a and rearrange: $NPV = CF/r$.

Now consider an investor holding s_2 dollars' worth of the shares of firm 2, representing a fraction α of the total outstanding stock, S_2. So, the investment, αS_2, gives the investor a return of

$$(1) \quad Y_2 = (X - rD_2).$$

Now, suppose the investor sells his αS_2, worth of company 2 shares and buys an amount $s_1 = \alpha(S_2 + D_2)$ of the shares of company 1. He can do so by utilizing the amount αD_2 realized from the sale of his initial holding, and *borrowing an additional amount*, αD_2, on his own account. This gives him a fraction $s_1/S_1 = \alpha(S_2 + D_2)/S_1$ of the shares, and therefore earnings, of firm 1. Taking into account the interest payments on the personal debt, $r\alpha D_2$, the return, Y_1, is given by

$$(2) \quad Y_1 = \frac{\alpha(S_2 + D_2)}{S_2} X - r\alpha D_2 = \alpha \frac{V_2}{V_1} X - r\alpha D_2$$

Since in both cases ((1) and (2)) the same amount of money has been invested, in equilibrium both investments should give the same return, Y_1, Y_2. Comparing now (1) and (2) we see that as long as $V_2 > V_1$ we must have $Y_1 > Y_2$, so that it pays owners of firms 2's shares to sell their holdings, thereby lowering S_2 and hence V_2; and to acquire shares of firm 1, thereby raising S_1 and thus V_1. MM conclude therefore that levered companies cannot command a premium over unlevered companies because investors have the opportunity of putting the equivalent leverage into their portfolio directly by borrowing on personal account. The possibility to borrow on personal account is a crucial element in the proof of the theorem and has become known as 'homemade leverage'.

A similar line of reasoning is followed for the other possibility, namely that the market value of the levered firm V_2 is less than V_1 (MM 1958, p. 270).

It is important to realize, that the MM Propositions only hold in an ideal, perfect, world, which has become known as the MM world. MM implicitely or explicitly assumed that:

1. Capital markets are frictionless.
2. Firms can lend at the risk-free rate (riskless debt).
3. Individuals can also borrow and lend at the risk-free rate.
4. There are no costs to bankruptcy.
5. Firms only issue two types of claims: risk-free debt and (risky) equity.
6. All firms are assumed to be in the same risk class.

7. There are no taxes.
8. All cash flow streams are perpetuities (i.e. no growth).
9. Corporate insiders and outsiders have the same information (i.e. no signalling opportunities).
10. Managers always maximize shareholders' wealth and do not expropriate in any way other stakeholders of the company (i.e. no agency costs).
11. Contracts are complete and can always be enforced.

What happened between June 1958 and today is that each of these assumptions has been relaxed in order to study the effect of every single imperfection on the MM results. The driving force behind this 'theory development' is the gap between theory and practice. Especially with respect to MM 1 the gap was immense. All real world debt-equity ratios vary within a certain range of, let's say, 60% to 20% debt. In fact, MM themselves ended their 1958 article with inviting others to study the relaxation of assumptions: "These and other drastic simplifications have been necessary in order to come to grips with the problem at all.", followed by the curtain line of the paper "Having served their purpose they can now be relaxed in the direction of greater realism and relevance, a task on which we hope others interested in this area will wish to share." (MM 1958, p. 296).

Essentially, ever since June 1958, theoretical papers on capital structure have been concerned with relaxing one or more assumptions. The form it has taken in the last decade, however, is to introduce new techniques (e.g. game theory) and new imperfections (e.g. incomplete contracts), which only implicitly or without realising were assumed to be absent before. The first step, however, was taken by MM themselves in their 1958 paper, but corrected in MM (1963).

1.3. Modigliani and Miller with corporate taxes, theorem B

In MM (1958) the first imperfection was already introduced: corporate taxes (Modigliani and Miller made a technical correction in 1963). The important thing about corporate taxes is that interest payments are tax deductible. MM (1963) showed that when corporate taxes are included, the value of the levered firm is equal to the value of an unlevered firm plus the present value of the tax shields associated by debt: $V^L = V^U + \tau_c D$, where τ_c is the corporate tax rate. In this way the capital structure that maximizes the value of a firm consists of 100% debt. This result we call theorem B; a corner solution where full debt financing is optimal.

In 1963 MM again used an arbitrage argument, similar to the one in 1958, to prove 'Theorem B'. Following their own line of reasoning, their 1963 proof can be summarized as follows. Using the same notation as MM,

X is the (long-run average) earnings before interest and taxes generated by the currently owned assets of a given firm in some stated risk class, τ is the marginal corporate tax rate,
X^τ is the after-tax return,
R is the interest bill, equals rD in the 1958 notation, and
ρ^τ is the rate at which the market capitalizes the expected returns net of tax of an unlevered company in a certain risk class.

Then $X^\tau = (1 - \tau)(X - R) + R = (1 - \tau)X + \tau R$.

This suggests that the after-tax return consists of two components: (1) an uncertain stream $(1 - \tau)X$; and (2) a sure stream τR. Therefore, the equilibrium market value of the combined stream can be found by capitalizing each component separately. More precisely, the market capitalizes the expected returns net of tax of an unlevered company in a certain risk class at rate ρ^τ, i.e.

$$\rho^\tau = (1 - \tau)X / V^u \quad \text{or} \quad V^u = (1 - \tau)X / \rho^\tau .$$

And since r is the rate at which the market capitalizes sure streams, r is the appropriate discount rate for the tax shield τD.
Then we would expect the value of levered firm with a permanent level of D to be

$$(3) \qquad V^L = \frac{(1-\tau)X}{\rho^\tau} + \frac{\tau R}{r} = V^U + \tau D$$

(MM 1963, p. 436-7).

Modigliani and Miller show that if (3) does not hold, investors can choose a more profitable portfolio by switching from relatively overvalued to relatively undervalued firms.
Suppose first that unlevered firms are overvalued, i.e. $V^L - \tau D < V^U$. An investor holding m dollars of stock in the unlevered firm has a right to a fraction m/V_U of the eventual return, i.e. $Y_U = (m/V_U)(1-\tau)X$.

Consider now an alternative portfolio obtained by investing m dollars as follows: the portion $(\{S_L / (S_L + (1 - \tau)D\}) \cdot m$ is invested in the stock of the levered firm, S_L, and the remaining portion, $(\{(1 - \tau)D / (S_L + (1 - \tau)D)\} \cdot m)$ is invested in its bonds (= debt). The stock component entitles the investor to a fraction $\{S_L / (S_L + (1 - \tau)D)\} \cdot m$ of the net profits of the levered firm, which equals $\{m / (S_L + (1 - \tau)D)\} \cdot \{(1 - \tau)(X - R)\}$. And the holding of the bonds yields $\{m / (S_L + (1 - \tau)D)\} \cdot \{(1 - \tau)R\}$. Hence the total return from the alternative portfolio is $Y_L = \{m / (S_L + (1 - \tau)D)\} \cdot \{(1 - \tau)X\}$ and this will dominate the uncertain income Y_U if, and only if, $S_L + (1 - \tau)D \equiv S_L + D - \tau D \equiv$ $\equiv V_L - \tau D < V_U$.

Thus, in equilibrium, V_U cannot exceed $(V_L - \tau)D_L$, for if it did investors would have an incentive to sell shares in the unlevered company and purchase the shares (and bonds) of the levered company.

A similar line of reasoning is followed for the other possibility, namely that the market value of the levered firm, $(V_L - \tau)D$, is less than the value of the unlevered firm, V_L. (See MM (1963), p. 427-8).

Theorem B was even more irrealistic than MM without taxes. There exists not a single firm which is voluntarily financed with 100% debt. Therefore the process of relaxation went on and in the beginning participants in the research programme were mainly looking for disadvantages of debt financing in order to come up with an internally optimal capital structure.

From MM theory it follows directly that the debt/equity ratio is irrelevant for the value of the firm, no matter the risk class. Hence assumption 6 was relaxed without theorem A or B being effected.

The factor that, intuitively, one would guess will alter the 100% debt corner solution will probably be the relaxation of the risk free debt assumption. In real life, when leverage (debt financing) increases debt becomes more risky since the chance that not all debt obligations can be met increases and debtors will therefore ask for a higher interest rate. Consequently, one would maybe say, since more money has to be paid to debtholders, that the value of the firm will decline if the debt/equity ratio rises. However, the introduction of risky debt does *not* change the MM propositions; it has no impact on the value of the firm. Stiglitz (1969) first proved this result, using a state preference framework, and Rubinstein (1973) provided a proof, using a mean-variance approach. Therefore, the introduction of risky debt cannot, by itself, be used to explain the existence of an optimal capital structure with a debt-equity ratio between 0% and 100% (a so called "interior" solution).

1.4. Bankruptcy costs, theorem C (an interior solution)

As we have seen, in a world without transactions costs risky debt does not affect on the value of the firm. However, when bankruptcy costs are taken into account, things are beginning to look differently.[10] Baxter (1967) was one of the first to suggest the existence of an internal optimal capital structure, based on bankruptcy costs: "If, ..., bankruptcy involves substantial administrative expenses and other costs, and causes a significant decline in the sales and earnings of the firm in receivership, the total value of the levered firm can be expected to be less than that of the all-equity company.". Since then, more sophisticated treatments have been offered by Kraus and Litzenberger (1973), Scott (1976) and Kim (1978).

When bankruptcy costs are considered, the value of the firm in bankruptcy is reduced by the fact that payments must be made to third parties other than bond- or shareholders. Trustee fees, legal fees, and other costs of reorganization or bankruptcy are deducted from the net asset value of the bankrupt firm and from the proceeds that should go to bondholders. Consequently, these "dead weight" losses associated with bankruptcy may cause the value of the firm in bankruptcy to be less than the discounted value of the expected cash flows from operations. This fact can be used to explain the existence of an interior optimal capital structure: theorem C.

1.5. Later developments

The next step in capital structure theory was the introduction of personal taxes (Miller 1977). Miller showed that, again, a "nothing matters" situation arises when you combine corporate and personal taxes. Since capital gains (equity income) are not taxed, but interest is taxed at the personal level, for the investor, who ultimately determines the market value of a company, there might even be a tax *disadvantage* to debt financing. Then, a new strand of literature was started by the famous Jensen and Meckling (1976) paper. They introduced the so called *agency theory* in the world of corporate finance, which relaxes the assumption of no conflict of interest between different parties, especially management, shareholders and debtholders. In particular, managers do not always act in the interest of the shareholders and consequently the goal is not always to maximize the value of the company. The paper shows that, based on these agency problems and without assuming taxes or

[10] It should be stressed that only bankruptcy *costs* but not bankruptcy *risk* can explain an optimal mix of debt and equity, because the bankruptcy *risk* is reflected in the higher interest rates when risky debt is introduced into the model.

bankruptcy costs, an optimal capital structure can be explained. One year later Ross (1977) introduced the existence of asymmetric information in capital structure theory. Assuming that managers have more information about the expected returns of the company than outside investors, he argued that greater financial leverage can be used by managers to signal an optimistic future of the firm. In the same year Leland and Pyle (1979) used the existence of asymmetric information to show that the firms value if positively related to the fraction of the owners stake in the company and therefore the firm will have greater debt capacity and use greater amounts of debt.

Since the late seventies, until the late eighties, virtually all research concerning capital structure issues has been concerned with agency and/or asymmetric informational issues. Since the middle of the eighties, interrelations between financing and investment decisions (e.g. Titman 1984) and capital structure choices in relation to takeovers (e.g. Harris and Raviv 1988) have been studied. Most recently, the assumption of comple contracts is relaxed. Instead, contracts are assumed to be oncomplete, i.e. they don't specify precise provisions for every conceivable future event (e.g. Cools and Zon 1992). And apart from the theoretical literature hundreds of papers try to empirically test all the different capital structure theories.[11]

Section 2

Structuralist reconstruction of the relevant theory

2.1. Interesting theorems

This section displays the logical skeleton of "A State-Preference Model of Optimal Financial Leverage" by Kraus and Litzenberger (1973). As usual in theoretical economics, the strategy of model construction is focused on an interesting theorem (Hamminga 1983). Here, what is to be derived is a theorem on how the value of (debt plus equity of) a firm depends on the proportion of debt in that value. Modigliani and Miller

[11] This started with a paper of Franco Modigliani and Merton Miller in the *American Economic Review*, June 1958.

(1958, 1963) used a plausible model to arrive at implausible, and therefore intriguing theorems: the proportion of debt has no influence at all on the value of the firm if corporate tax is nonexistent, and should be 100% if corporate tax exists. The model of Kraus and Litzenberger based on a state preferance approach, could explain the existence of an optimal solution between 0% and 100%.

2.2. *State preference*

The "state preference" approach consists of assuming a set J of all conceptually possible end-of-period "states of the world" j. In every single possible state of the world j there is a fixed and given return X_{fj} for every firm f, a given tax rate T_j and given bankruptcy cost C_{fj} ($C_{fj} = 0$ for firms f that survive in state j).

It is a one period analysis. The return (earnings before interest and taxes) X_{fj} of firm f in state j is identical to the end-of-period market value of firm f. If you would know what state j would be realized you would have no problems of uncertainty. Hence, the problem of uncertainty solely consists of investors not knowing what state will be realized, and having different probability beliefs concerning $j \in J$.

A "market expectation" about the states $j \in J$ is negotiated among investors by assuming tradeable "primitive securities" Π_j, which can be thought of as "lottery tickets" yielding 1$ if state j occurs and nothing if $j' \neq j$ occurs. The equilibrium price P_j of Π_j could be derived from 1) probability beliefs of investors concerning state j 2) their time preference for holding money, 3) their risk aversion and 4) their utility functions (Arrow 1969, Debreu 1959). Kraus and Litzenberger do not perform this derivation, but take P_j, $j \in J$ as given, exogenous variables. Any other security bought by an investor can now be identified with a definite number of lottery tickets Π_j for every state j (since investors are assumed to know the return of every security in every state).

2.3. *Potential models (conceptual framework)*

Now we are ready to specify wat structure a thing x should have in order to be a conceptual possibility in the language that Kraus and Litzenberger have chosen for their theory.

x is a *potential debt equity market system with corporate tax and bankruptcy cost* ("x is a *DETCp*") iff (if and only if)

$$x = \langle J, F, \Psi, \mathbf{R}, \Pi, D, X, T, C, P, Y, Z, B, S, V, \rangle$$

The symbols J, F, Ψ, $\mathbf{R}$, denote elementary sets. Their meanings are

- J the set of states of the world j
- F the set of firms f in the market system
- Ψ the set of primitive securites
- $\mathbf{R}$ the set of real numbers

The symbols Π, D, X, T, C, P, Y, Z, B, S, V, denote functions:

$\Pi: J \rightarrow \Psi$ yields the primitive security Π_j that has a return of 1\$ if state j occurs, and 0\$ in any state $j' \neq j$.

$D: F \rightarrow \mathbf{R}^+{}_D$ yields the debt of firm f, a promise to pay a fixed amount D_f, irrespective of the state that occurs, a nonnegative real number

$X: F \times J \rightarrow \mathbf{R}^+{}_X$ yields the return of f in state j, the end of period value of f, a real number X_{fj}, negative in case of bankruptcy.

$T: J \rightarrow \mathbf{R}_T{}^{[0,1]}$ yields the tax rate over X in state j, a real number T_j in the closed interval [0,1].

$C: F \times J \rightarrow \mathbf{R}^+{}_C$ is the bankruptcy cost C_{fj} of f in state j, zero if the firm survives in state j.

With the help of the primary functions Π, D, X, T, C, the secondary functions P, Y, Z, B, S, V, are defined.

$P: \Psi \rightarrow \mathbf{R}^+{}_P$ yields the price P_j of primitive security Π_j

$Y: F \times J \times \mathbf{R}^+{}_D \times \mathbf{R}_X \times \mathbf{R}^+{}_C \rightarrow \mathbf{R}^+{}_Y$ is the return to holders of debt D_f of every firm f in state j, where

$$Y_{fj}: \begin{cases} D_f \text{ for } D_f \leqslant X_{fj} \text{ (the firm "survived")} \\ X_{fj} - C_{fj} \text{ for } D_f > X_{fj} \text{ (the firm is bankrupt)} \end{cases}$$

$Z: F \times J \times \mathbf{R}^+{}_D \times \mathbf{R}_X \times \mathbf{R}_T[0,1] \rightarrow \mathbf{R}^+{}_Z$ is the return to holders of equities of f in state j, where

$$Z_{fj}: \begin{cases} X_{fj}(1-T_j) + T_j D_f - D_j \text{ for } D_f \leqslant X_{fj} \text{ (the firm "survived")} \\ 0 \text{ for } D_f > X_{fj} \text{ (the firm is bankrupt)} \end{cases}$$

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[12] Wolfgang Balzer correctly suggests in recent correspondence that the types of the functions Y, Z, B, S and V can be simplified to functions from $F \times J$ into the real numbers.

The last three functions have again a different logical role:

B: $F \times J \times \mathbf{R}^+{}_Y \times \mathbf{R}^+{}_P \times \mathbf{R}^+{}_D \times \mathbf{R}_X \times \mathbf{R}[0,1]_T \times \mathbf{R}^+{}_C \to \mathbf{R}^+{}_B$ yields the present market value of the debt D_f of firm f

S: $F \times J \times \mathbf{R}^+{}_Z \times \mathbf{R}^+{}_P \times \mathbf{R}^+{}_D \times \mathbf{R}_X \times \mathbf{R}[0,1]_T \times \mathbf{R}^+{}_C \to \mathbf{R}^+{}_S$ yields the equities of firm f

V: $F \times J \times \mathbf{R}^+{}_S \times \mathbf{R}^+{}_B \times \mathbf{R}^+{}_P \times \mathbf{R}^+{}_D \times \mathbf{R}_X \times \mathbf{R}[0,1]_T \times \mathbf{R}^+{}_C \to \mathbf{R}^+{}_B$ yields the present total value of the firm, where $V_f \equiv S_f + B_f$

Since B and S have only their domains and ranges specified, nothing is yet said about how they could be determined from the variables introduced earlier. This is all we do to specify the set *DETCp*. A thing x is a *potential* structure of the Kraus and Litzenberger theory, or a potential model iff it satisfies all requirements mentioned thusfar, and hence B_f, S_f and V_f can be, in an $x \in DETCp$, *any* nonnegative real number.

2.4. Models: introduction of the axioms on market forces

The set of models of the theory differs from the set of potential models by a specification of *how the present market value of debt and equity is determined*. This yields a subset $DETC \subset DETCp$ of structures:

x is a debt equity market system with corporate tax and bankruptcy cost (*DETC*) iff:

1) x is a *DETCp*.

2) P is such that primitive securities in $\Psi(\Pi_j, j \in J)$ have market equilibrium prices $P_j (j \in J)$. (An explicit definition of this as a defined function would require extension of *DETCp* with a set of investors, and functions mapping their probability beliefs on state $j \in J$, risk aversions, time preference and utility functions).

For convenient notation of the requirements on the functions B and S, Kraus and Litzenberger (re)number the states $j_1, j_2, ..., j_n$ in J for every firm in climbing order of the known return X_{fj} in each state. So we have, for every f:

$$X_{f1} \leqslant X_{f2} \leqslant ... \leqslant X_{fj} \leqslant ... \leqslant X_{fn}$$

3) B is such that for all firms f:

$$B(D) = \sum_{j=1}^{n} Y_{fj}P_j = \begin{cases} D_j \sum_{j=1}^{n} P_j \text{ iff } D_f \text{ is such that } D_f \leqslant X_{f1} \\ \sum_{j=1}^{k-1} (X_{fj} - C_{fj})P_j + D_j \sum_{j=k}^{n} P_j \text{ iff } D_f \text{ is such that} \\ \qquad \exists k > 1 \; (X_{f,k-1} < D_f < X_{fk}) \\ \sum_{j=1}^{n} (X_{fj} - C_{fj})P_j \text{ iff } D_f \text{ is such that } D_f > X_{fn} \end{cases}$$

The first of these three functions is meant for the case in which debt D_f is such that the firm survives (will not go bankrupt) in *every* possible state. The second function is meant for the cases where debt is such that the firm is bankrupt in state $j = 1, .., k-1$ and survives in the remaining states. The third function is meant for the case in which debt is such that in *all* states the firm will be bankrupt.

The present market value of debt is thus reduced to the present market value of primitive securities. The same is done for the equity of f:

4) S is such that for all f:

$$S(D) = \sum_{j=1}^{n} Z_{fj}P_j = \begin{cases} \sum_{j=1}^{n} [X_{fj}(1-T_j) + T_jD_f - D_f]P_j \text{ iff } D_f \text{ is such that } D_f \leqslant X_{f1} \\ \sum_{j=k}^{n} [X_{fj}(1-T_j) + T_jD_f - D_f]P_j \text{ iff } D_j \text{ is such that} \\ \qquad \exists k > 1 \; (X_{f,k-1} < D_f < X_{fk}) \\ 0 \text{ iff } D_f \text{ is such that } D_f > X_{fn} \end{cases}$$

The meaning of the tripartition is again bankruptcy in no/some/all possible end-of-period states.

2.5. *Interesting theorems: their exact shape*

This ends the list of additional requirements that a structure $x \in DETCp$ must meet in order to be an element of $DETC \subset DETCp$. The list is the result of a strategy meant to find the class of structures for which the following theorem *T-DETC* holds:

T-DETC: in all structures $x \in DETC$, for all firms f, if T_j is identical in all states j (dropping subscript j),

$$V(D_f) = V(0) + T \cdot B(D_f) - (1-T) \cdot \begin{cases} 0 \text{ iff } D_f \text{ is such that } D_f \leqslant X_{f1} \\ \sum_{j=1}^{k-1} C_{fj} P_j \text{ iff } D_f \text{ is such that } \\ \qquad \exists k > 1 \; (X_{f,k-1} < D_f < X_{fk}) \\ \sum_{j=1}^{n} C_{fj} P_j \text{ iff } D_f \text{ is such that } D_f > X_{fn} \end{cases}$$

Figure 1 depicts an example of a shape that $V(D_f)$ could take for some firm f:

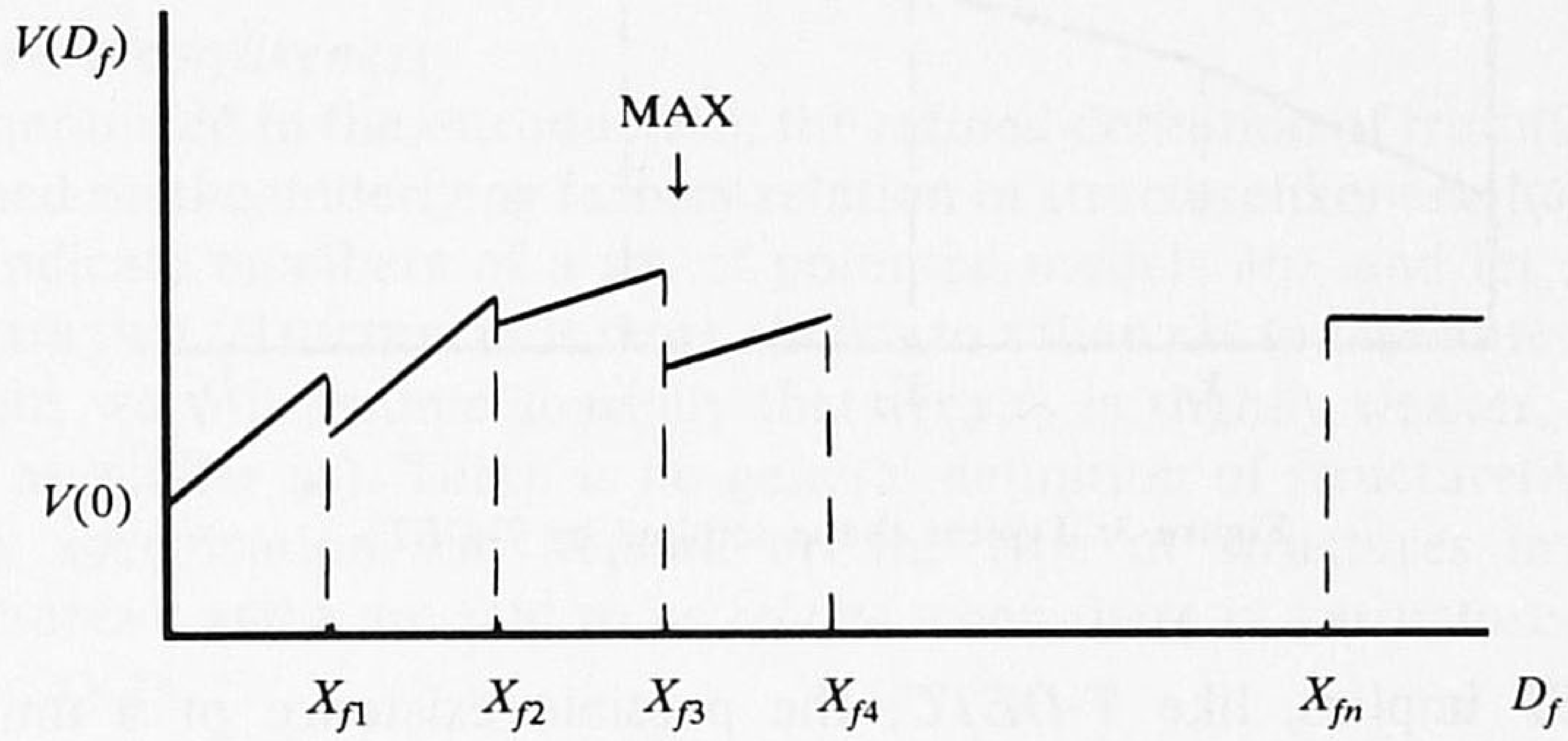

Figure 2: Typical shape implied by *T-DETC*

The slopes of the line segments are determined by T and P_j (Kraus and Litzenberger 1973, p. 916), and every time D_f passes the value at which it equals some state's total return of the firm, X_{fj}, the bankruptcy costs of that state kick down the value of the function $V(D_f)$.

T-DETC implies 1) that there *can* be a unique optimal amount of debt D_f, 2) this optimum amount of debt can be "interior", that is, larger than zero and smaller than the total return of the firm in the firm's most lucrative state (X_{fn}). Figure 2 is drawn as an example of this possibility.

This possibility arises as a result of the introduction of bankruptcy cost in the model. If bankruptcy costs are removed from the structures in *DETC* and *DETCp*, a poorer set of structures *DET*(p), results, where $x \in$ *DET*(p) is called *(potential) debt equity market system with corporate tax.* This brings us back to the original paradoxical Modigliani-Miller theorem on the optimal amount of corporate debt in case of the existence of corporate tax; setting $C_{fj} \equiv 0$ in *T-DETC* immediately yields *T-DET*:

T-DET: in all structures $x \in DET$, for all firms f, if T_j is identical in all states j:

$$V(D_f) = V(0) + T \cdot B(D_f)$$

This is the typical shape of $V(D_f)$:

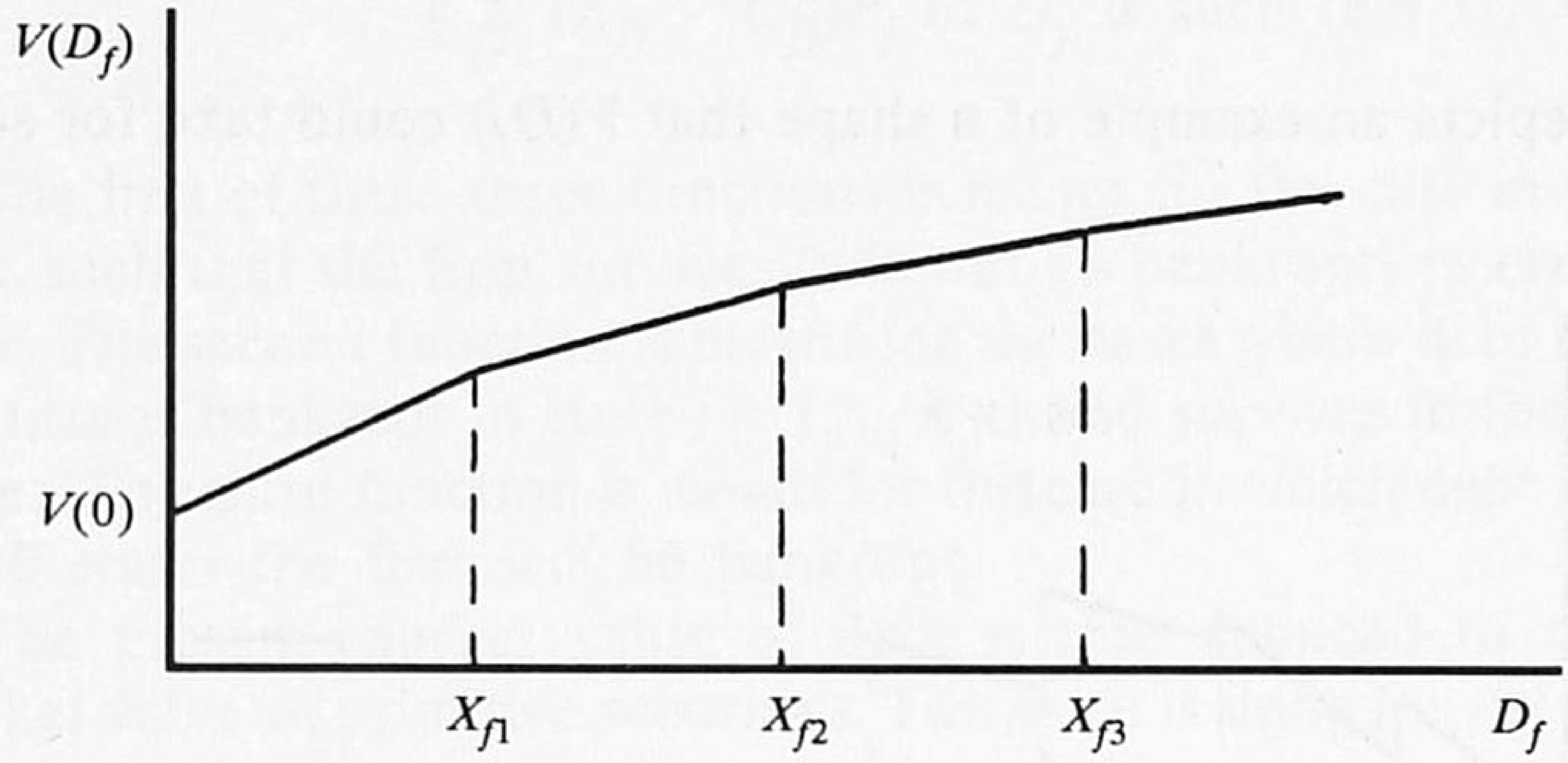

Figure 3: Typical shape implied by *T-DET*

T-DET implies, like *T-DETC*, the possible existence of a unique optimal amount of Debt D_f but if it exists, it is a "corner solution": the more debt, the higher the value of the firm, therefore 100% debt financing is optimal.

If, finally, tax T_j is removed from the structures, we obtain $DE(p)$, where $x \in DE(p)$ is called a (*potential*) *debt equity market system*. This yields the initial Modigliani-Miller Theorem *T-DE*.

T-DE: in all structures $x \in DE$, for all firms f, $V(D_f) = V(0)$

This means that the debt-equity ratio is irrelevant to the value of a firm. The graph becomes a horizontal line.

History would have been nice and simple if Modigliani and Miller had first discovered *T-DE* in the Kraus and Litzenberger framework presented above, then would have introduced tax to arrive at *T-DET*, and finally that Kraus and Litzenberger enriched the theory with bankruptcy cost to arrive at *T-DETC*.

However, Modigliani and Miller did not use the state preference framework. For what this example is meant to illustrate below, we do not need to go into the logical intricacies of reducing Modigliani and Miller's of introducing market forces to the method used by Kraus and Litzenberger.

Section 3
What the example illustrates: approximation of a provable interesting truth by concretization

3.1. Refined theorylikeness and concretization

Refined theorylikeness

As announced in the introduction, the refined definition of truthlikeness is based on the underlying ternary relation of structurelikeness. Let x, y, z etc. indicate members of a set of potential models Mp, and let $s(x,y,z)$ indicate that (structure) y is more similar to z than x is to z. (For technical reasons we will assume formally that $s(x,y,z)$ is slightly weaker, viz. at least as similar as). There is no general definition of structurelikeness. Every specification will depend on the type of structures involved. Structures x and z are said to be *related* when there is a structure y such that $s(x,y,z)$.

For what follows we only have to assume that s satisfies some very plausible minimal requirements:

centered: $s(x,x,x)$
centering: $s(x,y,x)$ implies $x = y$
conditionally reflexive: $s(x,y,z)$ implies $s(x,x,y)$, $s(x,x,z)$, $s(x,y,y)$, $s(x,z,z)$ and $s(y,z,z)$.

Refined *truth*likeness is conceived as a special case of a general relation of refined *theory*likeness (RTL) on the basis of s. For mutually non-overlapping subsets X, Y, Z of Mp, $RTL(X,Y,Z)$, Y is closer to Z than X, is defined by:

(i) for all x in X and z in Z if $r(x,z)$ there is y in Y such that $s(x,y,z)$
(ii) for all y in Y there are x in X and z in Z such that $s(x,y,z)$.

Informally speaking, $RTL(X,Y,Z)$ says (i) for every Z-model Y can improve upon any X-model related to that Z-model, and (ii) every Y-model improves upon some related X- and Z-model.

For technical reasons the definition formally allows the extreme situation that also $RTL(Y,X,Z)$ is true. Note that $RTL(X,Y,Z)$ reduces to the naive definition in the introduction when s is trivial, i.e. when $s(x,y,z)$ if and only if $x = y = z$.

Concretization

One specific notion of structurelikeness concerns concretization. We start with the definition of the constitutive binary relation: y is a concretization of x (and x an idealization of y), indicated by $con(x,y)$, if and only if y transforms into x, directly or by a limit procedure, when one or more constants and/or functions in y uniformly assume the value 0. On the basis of con we define the relevant ternary relation simply as follows: $<x,y,z>$ is a concretization sequence, indicated by $ct(x,y,z)$, if and only if $con(x,y)$ & $con(y,z)$. It is easy to check that con is reflexive, antisymmetric and transitive, from which it follows immediately that ct satisfies the minimal requirements for a structurelikeness relation. Moreover, relatedness on the basis of ct trivially coincides with con.

Let $RTLct$ indicate RTL based on ct. It is also plausible to define the following binary relation of concretization between theories: Y is a concretization of X, indicated by

$CON(X,Y)$ if and only if for all y in Y there is x in X such that $con(x,y)$ and for all x in X there is y in Y such that $con(x,y)$.

For mutually non-overlapping sets the following general concretization theorem (GC-theorem) is now easy to prove: if $CON(X,Y)$ and $CON(Y,Z)$ and if Y is convex and mediating then $RTLct(X,Y,T)$. Convexity and mediation are properties which are frequently and easily satisfied by a theory. Y is *convex* is the property that y is in Y when $ct(x,y,z)$ and x and z belong to Y. Y is *mediating* is the property that if $con(x,z)$ and if there are y and y' in Y such that $con(x,y)$ and $con(y',z)$ then there is y'' in Y such that $con(x,y'')$ and $con(y'',z)$, i.e. $ct(x,y'',z)$. It is important to note that convexity and mediation are properties of Y, independent of Y's relation to X and Z.

3.2. Applications

Now we are in the position to apply the above general notions to the case of capital structure theory. We will present the applications in three rounds. In the first application round we concentrate on the relations between the three theories, conceived as sets of models. In the second round we focus on interesting theorems and their domains of validity. Finally, in the third round we will characterize the heuristic strategy and hypotheses of the researchers which are typical for the present case of truth approximation. Only in the last round the application of the GC-theorem is of substantial importance, but we will mention its application also in the previous rounds.

Concretization of theories

Recall from Section 2 that *DE*, *DET*, and *DETC* are subsets of the set *DETCp* of potential Debt Equity market system with corporate Tax and bankruptcy Costs satisfying the extra conditions 2, 3 and 4 of section 2.4. Moreover, in the case of *DE*-models the bankruptcy cost function *C* and the corporate tax function *T* are uniformly zero, whereas in the case of *DET*-models only *C* is uniformly zero, and in the case of *DETC*, *T* nor *C* are uniformly zero. Note that by trivial consequence *DE*, *DET* and *DETC* are mutually non-overlapping.

Let *z* be a *DETC*-model. It is easy to check that there is a (unique) *DET*-model *y* and a (unique) *DE*-model *z* such that *con*(*x*,*y*) and *con*(*y*,*z*), and hence *ct*(*x*,*y*,*z*). From this it is a small step to prove directly *RTLct*(*DE*,*DET*,*DETC*), i.e. *DET* is closer to *DETC* than *DE* on the basis of concretization. This result is also easy to obtain *via* the *GC*-theorem from the fact that *DET* is convex and mediating and the combination of *CON*(*DE*,*DET*) and *CON*(*DET*,*DETC*).

Concretization of interesting theorems

In the second application round we introduce the interesting theorems in Hamminga's sense (section 2.1) and their domain of validity. Recall that theorems *T-DE*, *T-DET*, and *T-DETC* were associated with *DE*, *DET* and *DETC*, respectively. Let the consequents of *T-DE*, *T-DET* and *T-DETC* be indicated by *CT-DE*, *CT-DET* and *CT-DETC*, respectively. Let *VCT-DE*, *VCT-DET* and *VCT-DETC* indicate the subsets of *DEp*, *DETp* and *DETCp* for which these consequents are respectively provable, to be called their respective domains of validity. (For technical reasons we restrict *VCT-DE* and *VCT-DET* to *DEp* and *DETp*, respectively, assuring that the three domains of validity under consideration are mutually non-overlapping; extension to *DETCp* (only) meets some technical complications.)

The first crucial result of Modigliani and Miller can now be rephrased as the proof that *DE* is a subset of *VCT-DE*, i.e. *CT-DE* is provable for *DE*-models.[13] Their second result amounts to the proof that *DET* is a subset of *VCT-DET*. Finally, the crucial result of Kraus and Litzenberg is the proof that *DETC* is a subset of *VCT-DETC*.

Let us call in general, assuming some set of potential models *Mp*, a theorem *T** a concretization of *T* if and only if the domain of validity of *T** in *Mp* is a concretization of that of *T*, i.e. *CON*(*VT*,*VT**). This relation between two theorems may well be provable without the above mentioned type of subset proofs, as in the present case. That is, it is easy

[13] This can be strictly proven in the exposition of Kraus and Litzenberger (1973).

to check that *CON*(*VCT-DE*, *VCT-DET*) and *CON*(*VCT-DET*, *VCT-DETC*), and hence, by the *GC*-theorem and noting that *VCT-DET* is convex and mediating, that *VCT-DET* is closer to *VCT-DETC* than *VCT-DE* on the basis of concretization. Again, the latter result, i.e. *RTLct*(*VCT-DE*, *VCT-DET*, *VCT-DETC*), is also easy to prove directly. Recall that we arrived already in the first round to the conclusion *RTLct*(*DE*, *DET*, *DETC*).

The heuristic strategy of Kraus and Litzenberg

Why makes concretization sense? What further goals were Kraus and Litzenberg aiming at? Let us start from the basic result of Modigliani and Miller to the effect that *DE* is a subset of *VCT-DE*. We will omit further reference to the intermediate result concerning *(T-)DET* and use as much verbal formulations as possible.

The more or less explicit heuristic strategy of Kraus and Litzenberg which brought them to *(T-)DETC* can be characterized as follows. Look for DE^* and $CT\text{-}DE^*$ such that DE^* is a convex and mediating concretization of *DE*, $CT\text{-}DE^*$ is a convex and mediating (interesting) concretization of *CT-DE* and DE^* is a subset of $VCT\text{-}DE^*$.

The motivation for this strategy can be given in terms of an unknown set of (potential) economically possible systems *EP* (*EPp*), including of course the actual ones *EA*. To be precise, the following heuristic hypotheses form 'the good reasons' for this strategy.

- H1 it is possible to formulate a set of potential economically possible systems *EPp* such that there is a unique subset *EP* characterizing the economically possible systems and a unique subset *EA* of *EP* characterizing the economically actual systems,
- H2 all relevant sets can be reformulated as subsets of *EPp*,
- H3 *EP* is a concretization of DE^*,
- H4 there is an interesting theorem *CT-EP* such that
  - (a) *VCT-EP* is a concretization of $VCT\text{-}DE^*$
  - (b) *CT-EP* is provably true for *EP*-systems, and hence provably true for *AE*-systems.

If these hypotheses are true and if DE^* and $CT\text{-}DE^*$ are in accordance with the explicit heuristic goals we may conclude that DE^* is closer to *EP* than *DE*, and that $VCT\text{-}DE^*$ is closer to *VCT-EP* than *VCT-DE*. In both cases the application of the *GT*-theorem is crucial, as long as we do not dispose of *EP* and *CT-EP*.

The four hypotheses may or may not be true. There does not seem to be a reason why they are unlikely to be true, except for the fact that economic reality is infinitely complex. This objection however can be conceded by assuming only that the hypotheses are approximately true in some intuitive sense.

The other crucial point is H3: *EP* will only be a concretization of *DE** if the relevant factor(s) introduced in the concretization of *DE* to *DE** have been accounted for in an adequate way. Of course, it is impossible to evaluate the adequacy of the concretization of *DE* to *DE** in a conclusive way. Hence, we have to rely on good reasons, which may in general be of empirical and/or theoretical nature, leading to empirical and/or theoretical concretization. The example of Van der Waals is a clear case of theoretical concretization followed by empirical support. For Kraus and Litzenberger, the reasons to engage in concretization were empirical doubts about the validity of the original M&M-result for real world situations in general. The reasons to believe in the adequacy of the concretization were only of theoretical economic nature.

We may summarize the foregoing as follows. By concretizing a set of models (i.e. a theory) and a provable interesting theorem about them, we try to approximate a provable interesting truth about economically possible, and hence about economically actual, systems. The heuristic hypotheses guaranteeing the success of this attempt may well be true, provided the concretization of the theory is not an empirically empty gesture. Besides being a (convex and mediating) concretization in the logico-mathematical sense it has to be plausible on theoretical and/or empirical grounds.

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